Analysis and Design of sheet piles

Sheet pile wall is a special type of retaining walls that are generally made of steel, timber and in most limited case of concrete structures. Steel piles are the commonest because they can be used on all kinds of terrain, they can be used depth greater that 3 m, they are water-tight and can be re-used. Timber sheet piles are generally cheaper than steel sheet piles but can only be used for temporary structures where the depth of driving does not exceed 3m. Reinforced concrete sheet piles can only be used only when it is possible to jet them into fine sands or drive them into very soft soils. They are not suitable for tougher soils as they can generally break off under driving.

Types of sheet pile walls

Sheet pile walls are generally classified based on its structural form and loading system. Under these, we have:

1. The cantilever sheet pile walls and
2. The anchored sheet pile walls.

The cantilever sheet pile walls are further classified into: **free cantilever sheet pile walls** (cantilever sheet pile wall subjected to concentrated load at the top) – these derive stability entirely from the lateral passive resistance of the soil below the dredge level into which they are driven, **cantilever sheet pile walls** (these retains backfill at a higher level on one side) - the stability is entirely from the lateral passive resistance of the soil into which the sheet pile is driven.

The anchored sheet pile walls which are held above the driven depth by anchors provided at suitable level can be divided into two namely:

i. Fixed earth support piles, and
ii. Free earth support piles

Design of sheet piles

The deign of sheet pile walls lies on the determination of the DEPTH OF EMBEDMENT, d. There are unique equations for the design of cantilever sheet pile walls and anchored sheet pile walls. The equations also vary when the sheet pile walls are located in cohesionless soils against when they are located in cohesive soils (These can be assessed from specialized textbooks). The manual method of this design is tedious and I would show a manual approach of the design and simple software application to verify the design and which can be used to achieve the same purpose with ease. When the cantilever sheet pile walls are located in cohesionless soil, the depth of embedment calculated should be increased by 20% to 40% while for cantilever sheet pile walls located in clay soils, the calculated depth of embedment should be increased by 40% to 60%.

Design parts:

In the article, I would design the cantilever sheet pile wall without backfill using manual and software applications. Figure 1 shows a cantilever sheet pile in a cohesionless soil deposit.

The pole rotates about the point P. The pressure above P is passive in the front and active on the back side. However, the pressures below the point P are reversed, that is, there is active pressure in the front and passive on the backside. Figure 2 shows the actual pressure distribution. As the analysis using the actual pressure distribution is quite complicated, the pressure distribution is generally simplified as shown in Figure 3.

The depth, **a** of the point, P of the zero pressure is given by $p_1 - \gamma a (K_p - K_a) = 0$ » $a = p_1 / \gamma (K_p - K_a)$

Let the total active pressure above point P be P_1 acting at a height, Z_1 above P. The passive pressure is given by the diagram, PDE. The passive pressure intensity at the bottom tip A can be expressed as

$p_2 = \gamma (K_p - K_a)(d - a) = \gamma (K_p - K_a) b$ where **b** = d – a, in which d is the depth of point A below the dredge level.

The passive pressure is indicated by the diagram EAF on the back side. The intensity of pressure at the tip A is given by:

$p_3 = \gamma (h + d) K_p - \gamma d K_a$ » $\gamma (h + d) K_p - \gamma (b + a) K_a$

From the equation of equilibrium in the horizontal direction, $P_1 + P_3 - P_2 = 0$

The total pressure P_3 and P_2 can be expressed in terms of p_3 and p_2 as follows:

$P_1 + ½ m (p_2 + p_3) - ½ p_2 b = 0$ (1)

Equivalence area diagrams are shown below

From Eqn (1), $m = (½ p_2 b - P_1)/ ½ (p_2 + p_3) = (p_2 b - 2P_1)/ (p_2 + p_3)$ (2)

Taking moment of all forces about A,

$P_1 (b + z_1) - ½ p_2 b (b/3) + ½ m (p_2 + p_3) \times (m/3) = 0$ (3)

Substitute equation (2) into (3)

$P_1 (b + Z_1) - (p_2 b^2/6) + (p_2 + p_3)/6 [(p_2 b - 2P_1)/ (p_2 + p_3)]^2 = 0$ (4)

Eqn (4) can be re-written as

$b^4 + C_1 b^3 - C_2 b^2 - C_3 b - C_4 = 0$ (5)

$C_1 = p_4/ (\gamma (K_p - K_a))$; $C_2 = 8P_1/ (\gamma (K_p - K_a))$; $C_3 = [6P_1 [2\gamma (K_p - K_a) Z_1 + p_4]]/ (\gamma (K_p - K_a))^2$; $C_4 = [P_1 [6Z_1 p_4 + 4P_1]]/ (\gamma (K_p - K_a))^2$

In which, $p_4 = \gamma h K_p + \gamma a (K_p - K_a)$

Eqn (5) is solved by trial and error method to determine b, then d = b + a.

The depth, d is for a factor of safety of unity. The required depth D is usually taken as D = 1.2d to 1.4d. This gives a factor of safety of about 1.5 to 2.0.

Alternatively, a factor of safety can be applied to the passive resistance. In that case, the value of K_p is usually taken as 1/2 to 2/3 of the normal value while computing b from Eqn (5) and the required depth D is taken as equal to d.

In the above discussions, the depth of water table is not considered. If the water table on the front side is at same level as on the rear side, the analysis remains unaltered except that the submerged unit weight (γ^1) should be used for the soil below the water table. However, if the difference in the two levels is greater than 1m, the pressure due to water on the sheet pile should be found from the flow net and properly accounted for in the analysis.

Example

Determine the required depth of embedment or penetration for the cantilever sheet pile wall shown in the Figure below. Take $\gamma = 18$ kN/m^3; $\phi = 38^o$; h = 7m

Solution

$K_a = \tan^2 (45 – 38/2) = 0.238$
$K_p = \tan^2 (45 + 38/2) = 4.204$
$p_1 = 0.238 \times 18 \times 7 = 29.988 \text{ kN/m}^2$
$a = p_1/ (\gamma (K_p – K_a)) = 29.988/ (18 \times (4.204 – 0.238)) = 29.988/ 71.388 = 0.42$ m
$P_1 = ½ \times 29.988 \times 7 + ½ \times 29.988 \times 0.42 = 104.958 + 6.297 = 111.255$ kN
Taking moment about P and dividing by P_1
$Z_1 = [(104.958 \times 2.753) + (6.297 \times 0.28)]/ 111.255 = (288.949 + 1.7632)/111.255 = 2.613$m
$p_2 = \gamma (K_p – K_a) (b) = 18 (4.204 – 0.238) b = 71.388 b$
$p_3 = \gamma (h + d) K_p – \gamma dK_a = \gamma (h + b + a) K_p – \gamma (b + a) K_a = 18 (7 + b + 0.42) 4.204 – 18 (b + 0.42) 0.238$
$p_3 = 75.672 (7.42 + b) – 4.284 (b + 0.42) = 561.486 + 75.672 b – 4.284 b – 1.799 = 559.687 + 71.388 b$
From Eqn (2), $m = (p_2 b – 2P_1)/ (P_2 + P_3) = [71.388 b – (2 \times 111.255)]/ (559.687 + 71.388b + 71.388b) = [71.388 b – (222.51)]/ (559.687 + 142.776 b)$
From Eqn (4), $P_1 (b + Z_1) – (p_2b^2/6) + (p_2 + p_3)/6 [(p_2b – 2P_1)/ (p_2 + p_3)]^2 = 0$ » $[111.255 (b + 2.613)] – (71.388b^3/6) + [(559.687 + 142.776 b)/6] \times [(71.388 b^2 – (2 \times 111.255))/ (559.687 + 71.388b + 71.388b)]^2$
» $111.255 b + 290.7 – 11.898 b^3 + (93.281 + 23.796 b) \times [(71.388 b^2 -222.51)/ (559.687 + 142.776 b)]^2$
This equation can be solved by suitable trial and error method. However, I would adopt the second approach to solve it because it is simple and conformable to the tool available for me to solve it.
Alternatively,
$b^4 + C_1b^3 – C_2b^2 – C_3b – C_4 = 0$
$C_1 = p_4/ (\gamma (K_p – K_a))$?
$C_2 = 8P_1/ (\gamma (K_p – K_a))$?
$C_3 = [6P_1 [2\gamma (K_p – K_a) Z_1 + p_4]]/ (\gamma (K_p – K_a))^2$?
$C_4 = [P_1 [6Z_1p_4 + 4P_1]]/ (\gamma (K_p – K_a))^2$?
$p_1 = \gamma a (K_p - K_a) = 18 \times 0.42 (4.204 – 0.238) = 29.983$
$p_2 = \gamma b (K_p – K_a) = 18 \times 7 (4.204 – 0.238) = 499.716$
$p_3 = \gamma (h + b + a) K_p – \gamma (b + a) K_a = 18 (7 + b + 0.42) 4.204 – 18 (b + 0.42) 0.238 = 75.672 (7.42 + b) – 4.284 (b + 0.42) = 561.486 + 75.672b – 4.284 b – 1.799 = 559.687 + 71.388 b$
$p_4 = \gamma h K_p + \gamma a (K_p – K_a) = 18 \times 7 \times 4.204 + 18 \times 0.42 (4.204 – 0.238) = 529.704 + 29.983 = 559.687$
$C_1 = p_4/ (\gamma (K_p – K_a)) = 559.687/ (18 \times 3.966) = 7.84$
$C_2 = 8P_1/ (\gamma (K_p – K_a)) = (8 \times 29.983)/ (18 \times 3.966) = 3.36$
$C_3 = [6P_1 [2\gamma (K_p – K_a) Z_1 + p_4]]/ (\gamma (K_p – K_a))^2 = [6 \times 29.983 [2 \times 18 (4.204 – 0.238) 2.613 + 559.687]]/ (18 \times 3.966)^2 = (179.898 (373.074 + 559.687))/ 5096.2465 = 167801.8384/5096.2465 = 32.927$
$C_4 = [P_1 [6Z_1p_4 + 4P_1]]/ (\gamma (K_p – K_a))^2 = [29.983 [(6 \times 2.613 \times 559.687) + (4 \times 29.983)]]/ (18 \times 3.966)^2 = (29.983 (8774.7728 + 119.932))/ 5096.2465 = 266689.934/5096.2465 = 53.331$

C_1 = **7.84**; C_2 = **3.36**; C_3 = **32.927**; C_4 = **53.331**

Therefore, $b^4 + C_1b^3 – C_2b^2 – C_3b – C_4 = 0$ » $b^4 + 7.84\ b^3 – 3.36\ b^2 – 32.927\ b – 53.331 = 0$

Solving by trial and error method, b = 2.5

Therefore, d = b + a = 2.5 + 0.42 = 2.92

D = 1.5 d = 1.5 x 2.92 = 4.38, say 4.4 m

After determining the depth of embedment, the pile is checked to ensure that it passes all tests and to ensure that the depth of embedment determined is very satisfactory. This can be done with Tekla Tedds software. In a situation where the sheet pile has a surcharge load, it is also added and the depth of embedment investigated. In Tekla Tedds, water Table can be added, soil properties can be defined, cohesionless or cohesive soils can be defined and, steel section to be used can also be defined.

STEEL SHEET PILING ANALYSIS & DESIGN

In accordance with BS EN1997-1:2004 - Code of Practice for Geotechnical design and the UK National Annex

Geometry

Total length of sheet pile provided; H_{pile} = **11400** mm

Number of different types of soil; N_s = **1**

Retained height; d_{ret} = **6500** mm

Depth of unplanned excavation; d_{ex} = **500** mm

Total retained height; d_s = **7000** mm

Angle of retained slope; β = **0.0** deg

Loading

Soil characteristic properties table

Soil	ϕ'_k (deg)	δ_k (deg)	γ_m (kN/m^3)	γ_s (kN/m^3)	h (mm)
1	38.0	25.0	18.0	18.0	11400

Partial factors on actions - Section A.3.1 - Combination 1

Permanent unfavourable action; γ_G = **1.35**

Permanent favourable action; $\gamma_{G,f}$ = **1.00**

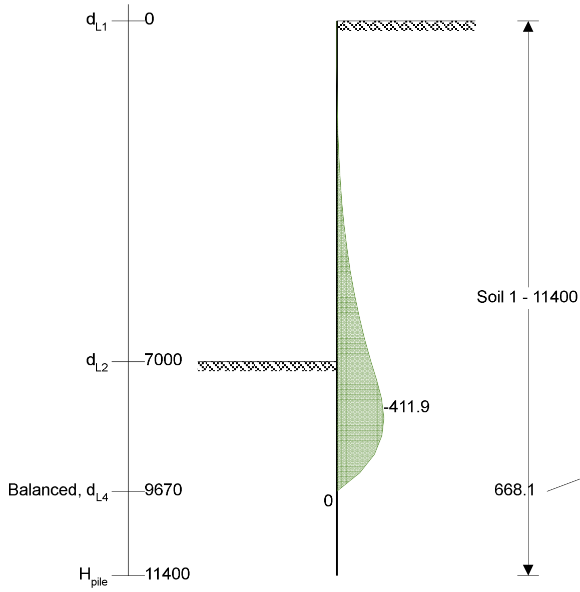
Variable unfavourable action; γ_Q = **1.50**

Angle of shearing resistance; $\gamma_{\phi'}$ = **1.00**

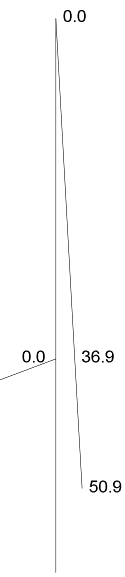
Weight density; γ_γ = **1.00**

Design properties table - combination 1

Soil	ϕ'_d	δ_d	$\gamma_{m.d}$	$\gamma_{s.d}$	K_a	K_p
1	38.0	25.0	18.0	18.0	0.217	13.901



SECTION THROUGH SHEET PILE WALL



BALANCE PRESSURE DIAGRAM (kN/m^2)

Overburden on active side

Overburden at 0 mm below GL in soil 1; OB'_{a11} = 0 kN/m^2 = **0.0** kN/m^2

Overburden at 7000 mm below GL in soil 1; OB'_{a21} =
$\gamma_G \times \gamma_{m.d1} \times h_{a1} + OB'_{a11}$ = **170.1** kN/m^2

Overburden at 9670 mm below GL in soil 1; OB'_{a31} =
$\gamma_G \times \gamma_{m.d1} \times h_{a2} + OB'_{a21}$ = **235.0** kN/m^2

Overburden on passive side

Overburden at 7000 mm below GL in soil 1; OB'_{p21} = 0 kN/m^2 = **0.0** kN/m^2

Overburden at 9670 mm below GL in soil 1; OB'_{p31} =
$\gamma_{G,f} \times \gamma_{m.d1} \times h_{p2} + OB'_{p21}$ = **48.1** kN/m^2

Pressure on active side

Active at 0 mm below GL in soil 1; $p'_{a11} = K_{a1} \times OB'_{a11}$ = **0.0** kN/m^2

Active at 7000 mm below GL in soil 1; $p'_{a21} = K_{a1} \times$
OB'_{a21} = **36.9** kN/m^2

Active at 9670 mm below GL in soil 1; $p'_{a31} = K_{a1} \times$
OB'_{a31} = **50.9** kN/m^2

Pressure on passive side

Passive at 7000 mm below GL in soil 1; $p'_{p21} = K_{p1} \times OB'_{p21}$ = **0.0** kN/m^2

Passive at 9670 mm below GL in soil 1; $p'_{p31} = K_{p1} \times$
OB'_{p31} = **668.1** kN/m^2

By iteration the depth at which the active moments equal the passive moments has been determined as 9670 mm as follows:-

Active moment about 9670 mm

Moment level 1; $M_{a11} = 0.5 \times p'_{a11} \times h_{a1} \times ((H - d_{L2}) + 2/3 \times h_{a1})$ = **0.0** kNm/m

Moment level 1; $M_{a12} = 0.5 \times p'_{a21} \times h_{a1} \times ((H - d_{L2}) + 1/3 \times h_{a1})$ = **645.6**
kNm/m

Moment level 2; $M_{a21} = 0.5 \times p'_{a21} \times h_{a2} \times ((H - d_{L3}) + 2/3 \times h_{a2})$ = **87.6** kNm/m

Moment level 2; $M_{a22} = 0.5 \times p'_{a31} \times h_{a2} \times ((H - d_{L3}) + 1/3 \times h_{a2})$ = **60.5** kNm/m

Passive moment about 9670 mm

Moment level 2; $M_{p21} = 0.5 \times p'_{p21} \times h_{p2} \times ((H - d_{L3}) + 2/3 \times h_{p2})$ = **0.0** kNm/m

Moment level 2; $M_{p22} = 0.5 \times p'_{p31} \times h_{p2} \times ((H - d_{L3}) + 1/3 \times h_{p2})$ = **793.8** kNm/m

Total moments about 9670 mm

Total active moment; ΣM_a = **793.7** kNm/m

Total passive moment; ΣM_p = **793.7** kNm/m

Required pile length

Length of pile required to balance moments; H = **9670** mm

Depth of equal pressure; d_{contra} = **7150** mm

Add 20% below this point; $d_{e\_add} = 1.2 \times (H - d_{contra})$ = **3023** mm

Minimum required pile length; $H_{total} = d_{contra} + d_{e\_add}$ = **10174** mm

Pass - Provided length of sheet pile greater than minimum required length of pile

Pile capacity (EN1993-5)

Maximum moment in pile (from analysis); $M_{pile} = \max(\text{abs}(M_{min}), \text{abs}(M_{max})) / 1\text{m}$ = **411.9** kNm/m

Maximum shear force in pile (from analysis); V_{pile} = **645.6** kN/m

Nominal yield strength of pile; f_{y_pile} = **270** N/mm^2

Name of pile; Arcelor AU25

Classification of pile; 2

Plastic modulus of pile; $W_{pl.y}$ = **2866** cm^3/m

Shear buckling of web (cl.5.2.2(6))

Width of section; $c = h / \sin(\alpha_{pile})$ = **551** mm

Thickness of web; $t_w = s$ = **10.2** mm

$\varepsilon = \sqrt{(235 \text{ N/mm}^2 / f_{y\_pile})}$ = **0.933**

$c / t_w = 54.1 = 57.9 \times \varepsilon <= 72 \times \varepsilon$

PASS - Shear buckling of web within limits

Bending

Interlock reduction factor (cl.5.2.2); β_B = **0.75**

Design bending resistance (eqn.5.2); $M_{c,Rd} = W_{pl.y} \times f_{y\_pile} \times \beta_B / \gamma_{M0}$ = **580.4** kNm/m

PASS - Moment capacity exceeds moment in pile

Shear

Projected shear area of web (eqn.5.6); $A_v = s \times (h - t)$ = **4442** mm^2

Design shear resistance (eqn.5.5); $V_{pl,Rd} = A_v \times f_{y\_pile} / (\sqrt{(3)} \times \gamma_{M0}) / b$ = **923.3** kN/m

PASS - Shear capacity exceeds shear in pile

Combined bending and shear

Shear presence mnt reduction factor (eqn.5.10); $\rho = (2 \times V_{pile} / V_{pl,Rd} - 1)^2$ = **0.159**

Reduced bending resistace (eqn.5.9); $M_{V,Rd} = \min((\beta_B \times W_{pl.y} - \rho \times A_v^2 / (4 \times s \times b \times \sin(\alpha_{pile}))) \times (f_{y\_pile} / \gamma_{M0}), M_{c,Rd})$ = **546.5** kNm/m

PASS - Reduced moment capacity exceeds moment in pile

Partial factors on actions - Section A.3.1 - Combination 2

Permanent unfavourable action; γ_G = **1.00**

Permanent favourable action; $\gamma_{G,f}$ = **1.00**

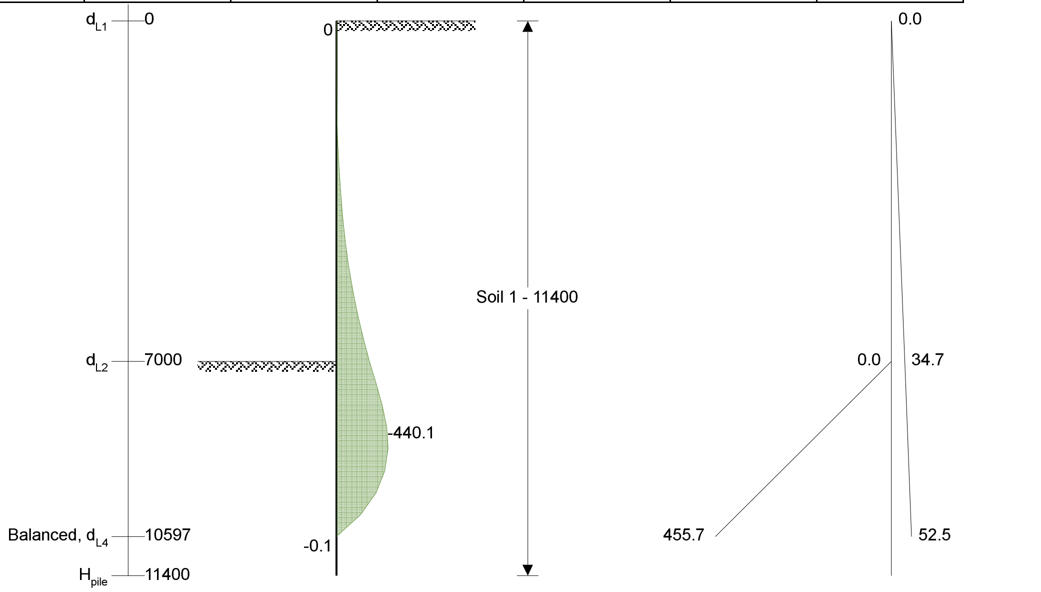
Variable unfavourable action; γ_Q = **1.30**

Angle of shearing resistance; $\gamma_{\phi'}$ = **1.25**

Weight density; γ_{γ} = **1.00**

Design properties table - combination 2

Soil	ϕ'_d	δ_d	$\gamma_{m.d}$	$\gamma_{s.d}$	K_a	K_p
1	32.0	20.5	18.0	18.0	0.275	7.038



Overburden on active side

Overburden at 0 mm below GL in soil 1; OB'_{a11} = 0 kN/m^2 = **0.0** kN/m^2

Overburden at 7000 mm below GL in soil 1; OB'_{a21} = $\gamma_G \times \gamma_{m.d1} \times h_{a1} + OB'_{a11}$ = **126.0** kN/m^2

Overburden at 10597 mm below GL in soil 1; OB'_{a31} = $\gamma_G \times \gamma_{m.d1} \times h_{a2} + OB'_{a21}$ = **190.7** kN/m^2

Overburden on passive side

Overburden at 7000 mm below GL in soil 1; OB'_{p21} = 0 kN/m^2 = **0.0** kN/m^2

Overburden at 10597 mm below GL in soil 1; OB'_{p31} = $\gamma_{G,f} \times \gamma_{m.d1} \times h_{p2} + OB'_{p21}$ = **64.7** kN/m^2

Pressure on active side

Active at 0 mm below GL in soil 1; $p'_{a11} = K_{a1} \times OB'_{a11}$ = **0.0** kN/m^2

Active at 7000 mm below GL in soil 1; $p'_{a21} = K_{a1} \times OB'_{a21}$ = **34.7** kN/m^2

Active at 10597 mm below GL in soil 1; $p'_{a31} = K_{a1} \times OB'_{a31}$ = **52.5** kN/m^2

Pressure on passive side

Passive at 7000 mm below GL in soil 1; $p'_{p21} = K_{p1} \times OB'_{p21}$ = **0.0** kN/m^2

Passive at 10597 mm below GL in soil 1; $p'_{p31} = K_{p1} \times OB'_{p31}$ = **455.7** kN/m^2

By iteration the depth at which the active moments equal the passive moments has been determined as 10597 mm as follows:-

Active moment about 10597 mm

Moment level 1; $M_{a11} = 0.5 \times p'_{a11} \times h_{a1} \times ((H - d_{L2}) + 2/3 \times h_{a1})$ = **0.0** kNm/m

Moment level 1; $M_{a12} = 0.5 \times p'_{a21} \times h_{a1} \times ((H - d_{L2}) + 1/3 \times h_{a1})$ = **719.9** kNm/m

Moment level 2; $M_{a21} = 0.5 \times p'_{a21} \times h_{a2} \times ((H - d_{L3}) + 2/3 \times h_{a2})$ = **149.6** kNm/m

Moment level 2; $M_{a22} = 0.5 \times p'_{a31} \times h_{a2} \times ((H - d_{L3}) + 1/3 \times h_{a2})$ = **113.2** kNm/m

Passive moment about 10597 mm

Moment level 2; $M_{p21} = 0.5 \times p'_{p21} \times h_{p2} \times ((H - d_{L3}) + 2/3 \times h_{p2})$ = **0.0** kNm/m

Moment level 2; $M_{p22} = 0.5 \times p'_{p31} \times h_{p2} \times ((H - d_{L3}) + 1/3 \times h_{p2})$ = **982.6** kNm/m

Total moments about 10597 mm

Total active moment; ΣM_a = **982.8** kNm/m

Total passive moment; ΣM_p = **982.8** kNm/m

Required pile length

Length of pile required to balance moments; H = **10597** mm

Depth of equal pressure; d_{contra} = **7285** mm

Add 20% below this point; $d_{e\_add} = 1.2 \times (H - d_{contra})$ = **3975** mm

Minimum required pile length; $H_{total} = d_{contra} + d_{e\_add}$ = **11259** mm

Pass - Provided length of sheet pile greater than minimum required length of pile

Pile capacity (EN1993-5)

Maximum moment in pile (from analysis); $M_{pile} = \max(\text{abs}(M_{min}), \text{abs}(M_{max}))$ / 1m = **440.1** kNm/m

Maximum shear force in pile (from analysis); V_{pile} = **541.3** kN/m

Nominal yield strength of pile; f_{y_pile} = **270** N/mm^2

Name of pile; Arcelor AU25

Classification of pile; 2

Plastic modulus of pile; $W_{pl.y}$ = **2866** cm^3/m

Shear buckling of web (cl.5.2.2(6))

Width of section; $c = h / \sin(\alpha_{pile})$ = **551** mm

Thickness of web; $t_w = s$ = **10.2** mm

$\varepsilon = \sqrt{(235 \text{ N/mm}^2 / f_{y\_pile})}$ = **0.933**

$c / t_w = 54.1 = 57.9 \times \varepsilon <= 72 \times \varepsilon$

PASS - Shear buckling of web within limits

Bending

Interlock reduction factor (cl.5.2.2); β_B = **0.75**

Design bending resistance (eqn.5.2); $M_{c,Rd} = W_{pl.y} \times f_{y\_pile} \times \beta_B / \gamma_{M0}$ = **580.4** kNm/m

PASS - Moment capacity exceeds moment in pile

Shear

Projected shear area of web (eqn.5.6); $A_v = s \times (h - t)$ = **4442** mm^2

Design shear resistance (eqn.5.5); $V_{pl,Rd} = A_v \times f_{y\_pile} / (\sqrt{(3)} \times \gamma_{M0}) / b$ = **923.3** kN/m

PASS - Shear capacity exceeds shear in pile

Combined bending and shear

Shear presence mnt reduction factor (eqn.5.10); $\rho = (2 \times V_{pile} / V_{pl,Rd} - 1)^2$ = **0.030**

Reduced bending resistace (eqn.5.9); $M_{V,Rd} = \min((\beta_B \times W_{pl.y} - \rho \times A_v^2 / (4 \times s \times b \times \sin(\alpha_{pile}))) \times (f_{y\_pile} / \gamma_{M0}), M_{c,Rd})$ = **574.0** kNm/m

PASS - Reduced moment capacity exceeds moment in pile